Evidence-Based Subjective Logic

Boris Škorić
Nicola Zannone
Sebastiaan de Hoogh

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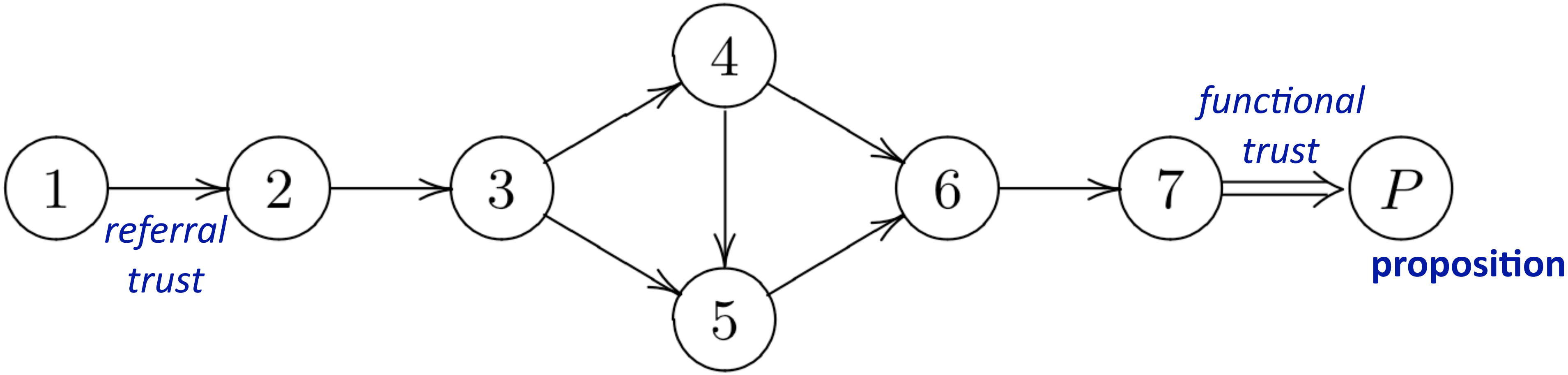
http://arxiv.org/abs/1402.3319

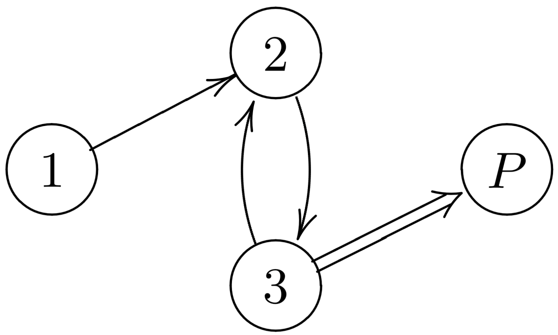
Outline

- Trust networks
- Forming opinions with uncertainty
  - Subjective Logic
  - ... hard to combine with Trust Networks
- Subjective Logic revisited
  - new chaining rule
  - evidence plays central role
- Trust networks with the new chaining rule

Trust Networks



Question: what should user 1 think about proposition P?



Subjective Logic

Opinion triplet $x = (x_b, x_d, x_u)$

belief

disbelief

uncertainty

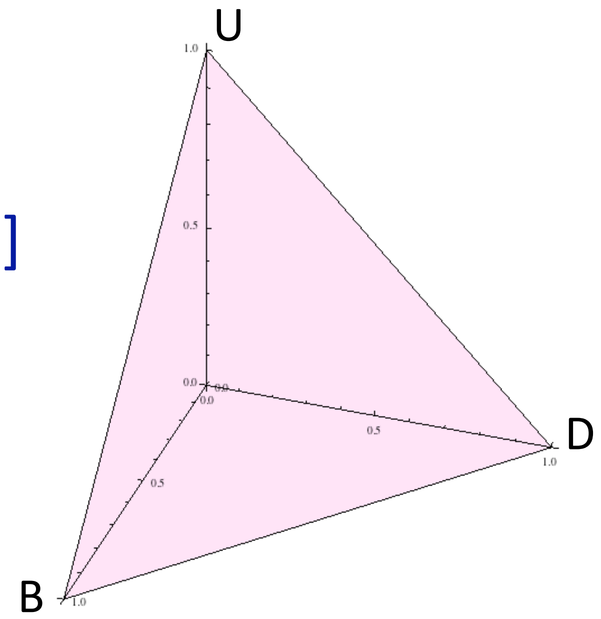
$x_b + x_d + x_u = 1$

Opinion about a proposition P, given some evidence E:

x_b = Prob[P can be proven | E]

x_d = Prob[P can be *dis*proven | E]

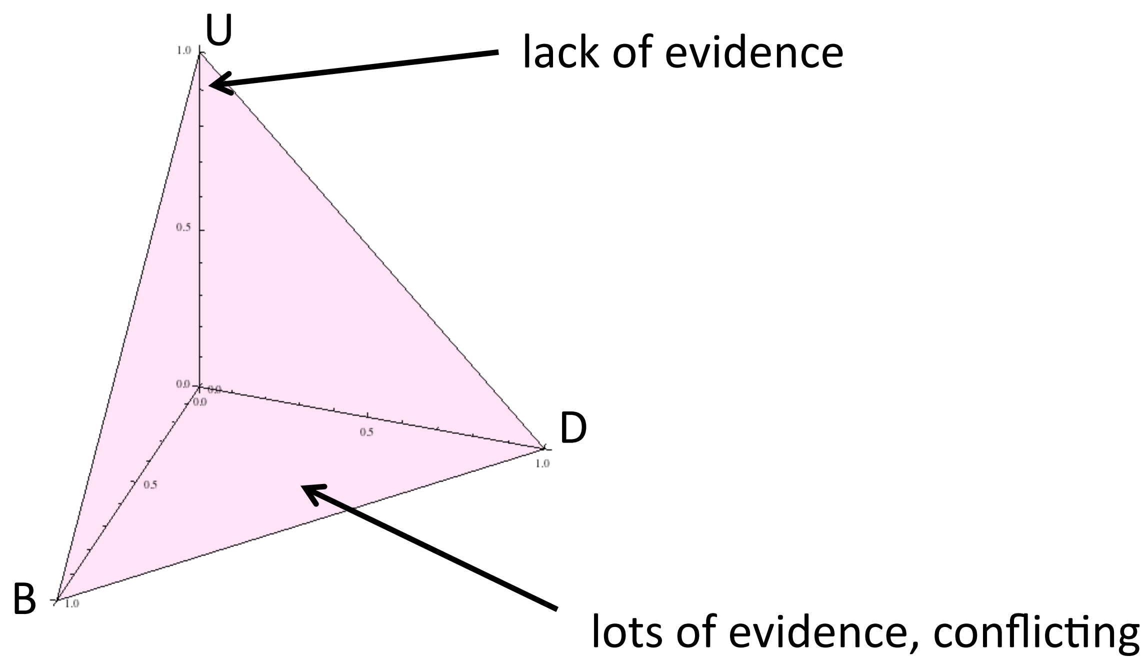
x_u = Prob[nothing can be proven about P | E]



Subjective Logic ≠ fuzzy logic

Subjective Logic distinguishes between

- conflicting evidence
- lack of evidence



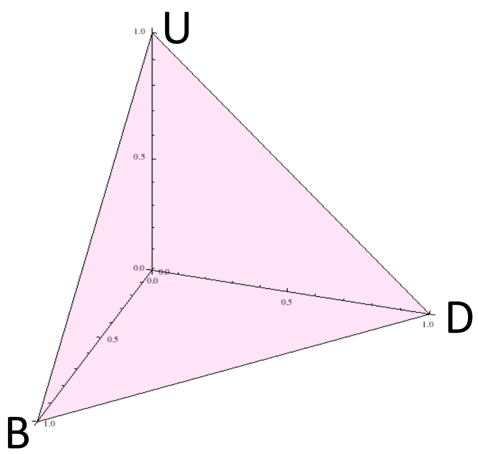
Relation between opinion and evidence

[Jøsang 2001]

p = amount of evidence supporting proposition P
n = amount of evidence refuting P

$$(x_\mathrm{b}, x_\mathrm{d}, x_\mathrm{u}) = \frac{(p, n, 2)}{p+n+2}; \quad (p, n) = \frac{2(x_\mathrm{b}, x_\mathrm{d})}{x_\mathrm{u}}$$

Strange constant;
comes from Beta distributions ...



Adding opinions: piling up evidence

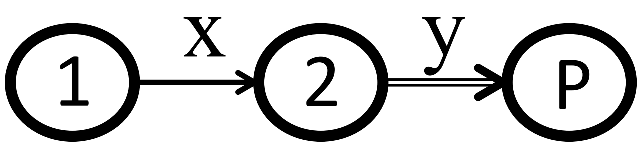
"Consensus" rule: Combining two opinions, x and y

$$x \oplus y \stackrel{\text{def}}{=} \frac{(x_{\mathrm{u}} y_{\mathrm{b}} + y_{\mathrm{u}} x_{\mathrm{b}}, x_{\mathrm{u}} y_{\mathrm{d}} + y_{\mathrm{u}} x_{\mathrm{d}}, x_{\mathrm{u}} y_{\mathrm{u}})}{x_{\mathrm{u}} + y_{\mathrm{u}} - x_{\mathrm{u}} y_{\mathrm{u}}}$$

p = p(x) + p(y)   n = n(x) + n(y)

Reduction of uncertainty

Chaining of opinions



"Discounting" rule:

$$x \otimes y \stackrel{\text{def}}{=} (x_{\mathrm{b}} y_{\mathrm{b}},\ x_{\mathrm{b}} y_{\mathrm{d}},\ x_{\mathrm{d}} + x_{\mathrm{u}} + x_{\mathrm{b}} y_{\mathrm{u}})$$

simple multiplication factor x_b

Associative: $x \otimes (y \otimes z) = (x \otimes y) \otimes z$

Problems with the chaining rule

- no interpretation in terms of evidence
  - pathological cases
- causes trouble in trust networks
  - double counting of evidence
  - topology-dependent formulas
  - cannot handle loops
    ("canonical form" needs loop removal)
- distributive property is missing,

$$x \otimes (y \oplus z) \neq (x \otimes y) \oplus (x \otimes z)$$

Underlying reason:

- Consensus is based on addition of **evidence**;
- Discounting based on **probability** multiplication.

Our contributions

- Relation between evidence and opinions
  - simpler theory
- Scalar multiplication rule
  - scales amount of evidence
- New chaining rule "$\boxtimes$"
  - evidence plays central role
  - $x \boxtimes (y \oplus z) = (x \boxtimes y) \oplus (x \boxtimes z)$
  - can deal with arbitrary Trust Networks

Opinions and evidence, revisited

Theorem 4.1: Let $p \geq 0$ be the amount of evidence that supports 'belief'; let $n \geq 0$ be the amount of evidence that supports 'disbelief'. Let $x = (b, d, u)$ be the opinion based on the evidence. If we demand the following properties,

1) $b/d = p/n$
2) $b + d + u = 1$
3) $p + n = 0 \;\Rightarrow\; u = 1$
4) $p + n \to \infty \;\Rightarrow\; u \to 0$

then the unique solution for x is

$$x = (b, d, u) = \frac{(p, n, c)}{p + n + c} \quad ; \quad (p, n) = c\frac{(b, d)}{u}$$

- c>0, more general than "2"
- "caution" parameter

Consensus operation does not depend on c

Scalar multiple of an opinion

Scalar α, opinion x

Definition of **α·x**:

p(α·x) = α p(x)   n(α·x) = α n(x)

$$\alpha \cdot x \stackrel{\text{def}}{=} \frac{(\alpha x_{\mathrm{b}}, \alpha x_{\mathrm{d}}, x_{\mathrm{u}})}{\alpha(x_{\mathrm{b}} + x_{\mathrm{d}}) + x_{\mathrm{u}}}$$

Nice linear properties,

$\alpha \cdot (x \oplus y) = (\alpha \cdot x) \oplus (\alpha \cdot y)$ and $(\alpha + \beta) \cdot x = (\alpha \cdot x) \oplus (\beta \cdot x)$

$$\alpha \cdot (\beta \cdot x) = (\alpha\beta) \cdot x$$

New chaining rule

Choose a function g that maps opinions to [0,1)

Definition: $x \boxtimes y = g(x) \cdot y$

[Scalar mult. of the evidence underlying y.]

Distribution property:

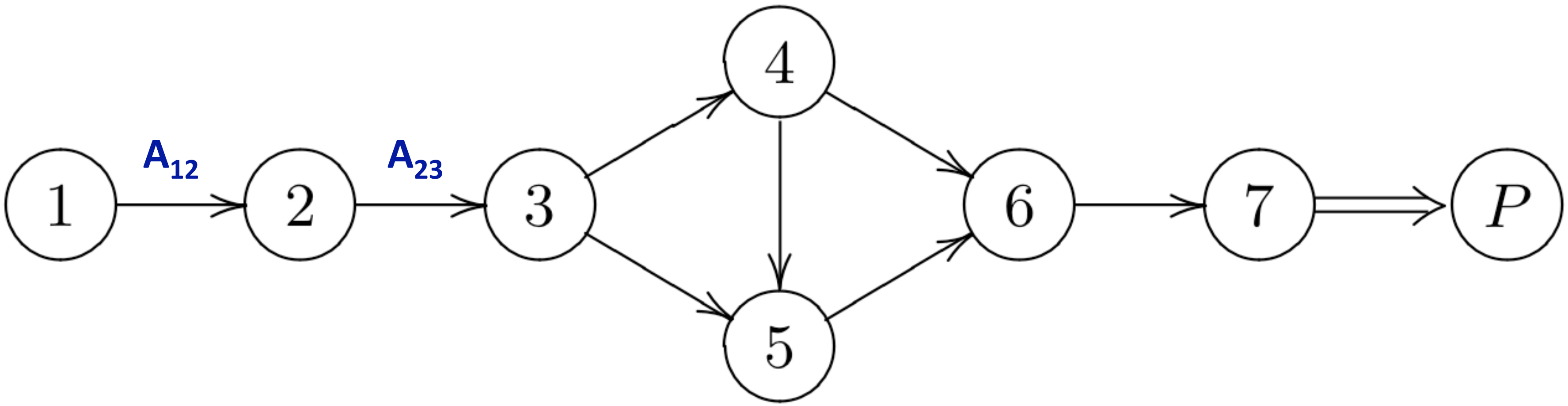
$$x \boxtimes (y \oplus z) = (x \boxtimes y) \oplus (x \boxtimes z)$$

But <u>not</u> associative.
(Not necessary!)

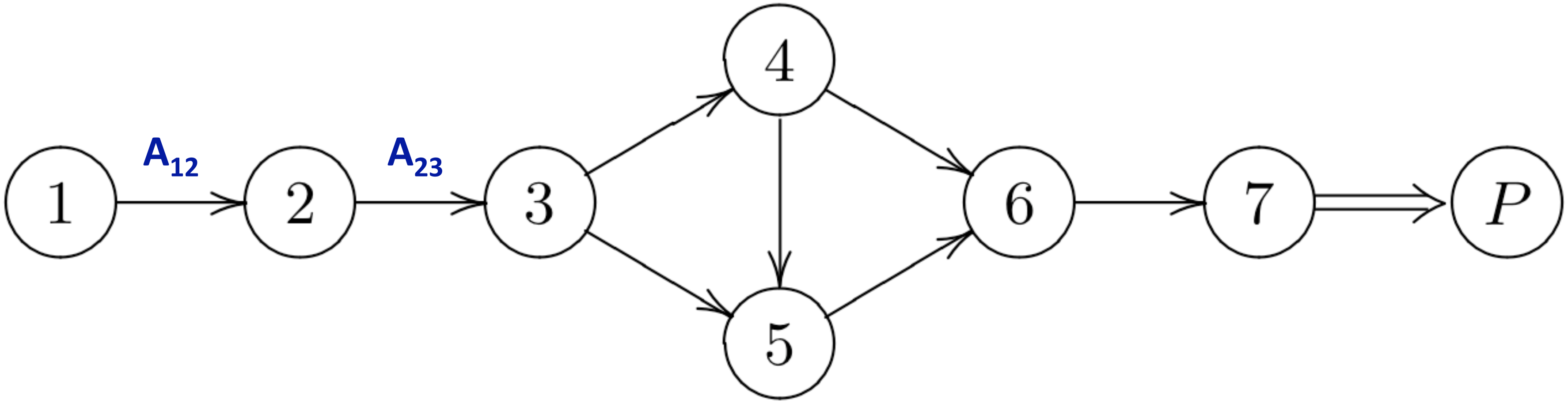
Trust networks with the new chaining rule

- "Lossy transport of evidence"
- Recursive solution R, given direct opinions A_{ij}:

$$R_{ij} = A_{ij} \oplus \bigoplus_{k:k\neq i} (R_{ik} \boxtimes A_{kj})$$



Example: $R_{16} = (R_{14} \boxtimes A_{46}) \oplus (R_{15} \boxtimes A_{56})$



$$
\begin{aligned}
F_{1P} &= R_{17} \boxtimes T_{7P} \\
R_{17} &= R_{16} \boxtimes A_{67} \\
R_{16} &= (R_{14} \boxtimes A_{46}) \oplus (R_{15} \boxtimes A_{56}) \\
R_{15} &= (R_{14} \boxtimes A_{45}) \oplus (R_{13} \boxtimes A_{35}) \\
R_{14} &= R_{13} \boxtimes A_{34} \\
R_{13} &= R_{12} \boxtimes A_{23} \\
R_{12} &= A_{12}.
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
R_{16} = {} & ((A_{12} \boxtimes A_{23}) \boxtimes A_{34}) \boxtimes A_{46} \oplus \\
& \left\{ ((A_{12} \boxtimes A_{23}) \boxtimes A_{34}) \boxtimes A_{45} \oplus (A_{12} \boxtimes A_{23}) \boxtimes A_{35} \right\} \boxtimes A_{56}
\end{aligned}
}
$$

Numerical experiments

	Flow-based	Flow-SL	SL (canonical form)	EBSL ($g(x) = x_b$)
C1	0.401	$(0.024, 0.220, 0.756)$	$(0.014, 0.123, 0.863)$	$(0.095, 0.859, 0.046)$
C2	0.392	$(0.003, 0.246, 0.751)$	$(0.002, 0.137, 0.861)$	$(0.011, 0.984, 0.005)$
C3	0.501	$(0.9993, 0.000, 0.0007)$	$(0.999, 0.000, 0.001)$	$(0.9998, 0.0000, 0.0002)$

Table 2 Comparison in terms of trust values r_{1P} and opinions F_{1P}

New chaining rule preserves more evidence

Summary

- Subjective Logic
  - good way of capturing uncertainty
  - addition rule $\oplus$ piles up evidence
  - "multiplication" rule $\otimes$ inconsistent with $\oplus$
    => all kinds of trouble
- Relation between evidence and opinions
- New: *scalar multiple of opinion*
  - scalar multiple of evidence
- **New chaining rule $\boxtimes$**
  - no longer associative
  - (right) distribution property
  - arbitrary Trust Networks can be handled;
    lossy transport of evidence